# Formulated Discrete Mathematical Model for Delayed Particle Flow in Cascaded Sub-Surface Water Reservoirs

Ombaki Richard[1], Kerongo Joash[2], Okwoyo James[3]

1. Department of mathematics and actuarial science- (Kisii University);
2. Department of mathematics and actuarial science- (Kisii University);
3. Department of mathematics and actuarial science- (University of Nairobi);

## Abstract

Pollution of sub-surface water reservoirs mainly rivers, streams, ponds and dams through contaminated water point sources (CWPS) was studied. The objectives were to formulate a discrete time delay mathematical model which describes the dynamics of reservoir pollution using mixing-problem processes that involve single species contaminants such as nutrients, pesticides, insecticides, herbicides and detergents. A conceptual perspective of mixing problem process in water tanks was applied to model delayed particle flow in cascaded water reservoir systems. The concentration ($x$) of pollutants was expressed as a function of the inflow and outflow rates using the principle for the conservation of mass. Systems of ODEs were generated from principles of mixing problems and then refined into a system of DDEs so that the concentration of pollutant leaving the reservoir at time $t$ would be determined at some earlier instant, $t-h$ for the delay $h>0$. The formulated model is a mathematical discrete time delay model which would be used to describe the dynamics of sub-surface water reservoir pollution. Solutions for the model were obtained numerically to determine the time necessary for pollutant movement in water reservoirs. The results from the validation of the model indicated that if the time delays in the mixing processes are not accounted for in particle movement in water reservoirs, and since mixing processes are never instantaneous, then the time required for transport of a species of pollutant in water reservoirs is underestimated.

**Key words**: Particle Flow, Sub-Surface Water, Reservoirs Pollution.