KIBABII UNIVERSITY COLLEGE (KIBUCO)

MAIN CAMPUS

UNIVERSITY EXAMINATIONS
2014 /2015 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER EXAMINATIONS

MAIN EXAMINATION

FOR THE DEGREE

OF

BACHELOR OF EDUCATION SCIENCE

COURSE CODE: STA 341

COURSE TITLE: THEORY OF ESTIMATION

DATE: Tuesday 13TH JANUARY, 2015 **TIME: 3.00-5.00 P.M**

INSTRUCTIONS TO CANDIDATES:

Answer Question ONE and any other Two Questions

TIME: 2 Hours

QUESTION 1:

(a) List four desirable classical properties which an estimator should posses. (4 marks)

(b) If $\overline{X}_1$ and $\overline{X}_2$ are the respective means of random samples of sizes n_1 and n_2 from a population with mean μ ; prove that an unbiased estimator of μ is given by

$$\hat{\mu} = \frac{n_1\overline{X}_1 + n_2\overline{X}_2}{n_1 + n_2}$$ (6 marks)

(c) The following random sample was obtained from a population with mean μ and variance, σ^2

12, 8, 11, 10, 8, 8, 3, 9. 11, 10

Obtain the unbiased estimates of μ and σ^2 (5 marks)

(d) Suppose that $E(\hat{\theta}_1) = E(\hat{\theta}_2) = \theta,\ , Var(\hat{\theta}_1) = \sigma_1^2$ and $Var(\hat{\theta}_2) = \sigma_2^2$

A new unbiased estimator of θ_3 is to be formed by

$$\hat{\theta}_3 = a\hat{\theta}_1 + (1-a)\hat{\theta}_2$$

How should the constant 'a' be chosen in order to minimize the variance of $\hat{\theta}_3$?

Assume that $\hat{\theta}_1$ and $\hat{\theta}_2$ are independent. (5 marks)

(e)Define the following terms as used in theory of estimation:

(i)a point estimate

(ii)a point estimator

(iii)a parameter

(2 + 2 + 1 marks)

(f) Let $X_1, X_2, \ldots\ldots.. X_n$ be a random sample drawn from an exponentially distributed population with parameter θ and probability density function given as

$$f(x) = \theta e^{-\theta x} \quad ,x > 0$$
$$= 0 \quad \text{elsewhere}$$

Show that the maximum likelihood estimator (MLE) for θ is $\frac{1}{\overline{X}} = \frac{n}{\sum_{i=1}^{n} X_i}$

(5 marks)

QUESTION 2 :

(a)Consider a simple linear regression model of the form :

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i \qquad \text{for i= 1,2, ...,n}$$

Where β_0 and β_1 are constants which are population parameters and ε_i is the model error

By minimizing the residual sum of squares, obtain the least squares estimators of β_0 and β_1. (10 marks)

(b)Let $\hat{\beta}_0$ and $\hat{\beta}_1$ be respectively the estimators of β_0 and β_1.Show that,

(i) $E(\hat{\beta}_0) = \beta_0$ (4 marks)

(ii) $E(\hat{\beta}_1) = \beta_1$ (4 marks)

Explain the importance of the results in (i) and (ii) above, in estimation theory.

(2 marks)

QUESTION 3:

(a)What is a consistent Statistic? (2 marks)

(b)Consider X_1, X_2,………. ,X_n to be a random sample of size n drawn from a population with unknown mean μ and variance σ^2 .If S^2 is the sample variance, show that,

$\dfrac{nS^2}{n-1}$ is a consistent estimator of σ^2 (8 marks)

(c)For a random sample of size n, X_1,X_2, ………X_n drawn from a population with mean μ and variance σ^2, show that

i)both $\frac{1}{2}(X_1 + X_2)$ and X_4 are unbiased estimators of μ (5 marks)

ii) $\frac{1}{2}(X_1 + X_2)$ is not a consistent estimator of μ (5 marks)

QUESTION 4 :

(a)Given a random sample $X_1, X_2, \ldots\ldots\ldots\ldots, X_n$ from a distribution having a probability density function, f(x; θ), $\theta \in \Omega$, illustrate how you would get the maximum likelihood estimator $\hat{\theta}$ for the population parameter θ . (6 marks)

(b)Suppose that a random sample of size n is drawn from a Bernoulli distribution whose probability mass function is

$$f(x) = \theta^{x}(1-\theta)^{1-x},,,,,,,x = 0,1; 0 \le \theta \le 1$$

Obtain the maximum likelihood estimator (MLE) for c (7 marks)

(c)Let $X_1, X_2, \ldots\ldots\ldots\ldots, X_n$ be a random sample from a normal distribution, N(θ ,1),

$-\infty < \theta < \infty$. Show that $\bar{X}$ is the MLE for θ (7 marks)

QUESTION 5:

(a)i)Define Sufficiency in relation to estimators (3 marks)

ii)State the factorization theorem of Fisher and Neyman pertaining to sufficiency

(4 marks)

iii) Let $X_1, X_2, \ldots\ldots\ldots\ldots, X_n$ be a random sample of size n from a Geometric distribution that has probability function

$$f(x) = \theta(1-\theta)^{x} \qquad \text{for } x=0,1,2,3,\ldots\ldots;\ 0<\theta<1$$

Show that $T = \sum_{i=1}^{n} X_i$ is a sufficient statistic for θ (6 marks)

(b) Let $X_1, X_2, \ldots\ldots\ldots\ldots, X_n$ be a random sample of size n from the negative exponential density,

$$f(x) = \theta e^{-\theta x},,,,,,,,,, for,, x > 0$$

$$= 0 \qquad \text{otherwise.}$$

Obtain the method of moments estimator for θ (7 marks)