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UNIVERSITY REGULAR EXAMINATIONS

2012 /2013 ACADEMIC YEAR

FOR THE DEGREE OF

BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: **MAT 204**

COURSE TITLE: **REAL ANALYSIS I**

DATE: 19th August 2013 **TIME**: 9.00 pm – Noon

INSTRUCTIONS

This paper consists of **TWO** sections; **A** and **B**. Answer **BOTH** questions in **SECTION A** and **ANY OTHER THREE** questions from **SECTION B**.

SECTION A

*Answer **BOTH** questions in this section.*

QUESTION 1 (16 marks)

a. Define the following terms;
   i. Monotone function   ii. Subsequence
   ii. Infimum of a set   iv. Function   (4 marks)

b. Prove by Mathematical Induction that for all natural numbers,

$$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + . . . + n^2 = \frac{1}{6} n(n + 1)(2n + 1)$$ (4 marks)

c. Suppose that $x_n \to l$ as $n \to \infty$ and that < x_{n_r} > is a subsequence of < x_n >. Prove that $x_{n_k} \to l$ as r $\to \infty$. (4 marks)

d. Prove that if the series $\sum_{n=1}^{\infty} a_n$ converges, then $a_n \to 0$ as $n \to \infty$.

(4 marks)

QUESTION 2 (15 MARKS)

a. Show that if x and y are positive, then x < y if and only if $x^2 < y^2$. (4 marks)

b. Draw a diagram illustrating the set of all(x, y) such that

$$Y = \begin{cases} 5 \; if \; x \geq 1 \\ 2 \; if \; x < 1 \end{cases}$$

Explain why this is a graph of a function from R to itself. What is the range of this function? What is the image of the set [1, 2] under this function? (3 marks)

c. Prove De Morgan's Law ; (A $\cup$ B)' = A' $\cap$ B'. (4 marks)

d. Consider the set {x: 2 $\leq$ x ¿ 3}. State the maximum, minimum, lub and glb if the exist. Is the set bounded? (3 marks)

SECTION B (39 MARKS)

Answer ANY THREE questions in this section.

QUESTION 3 (13 MARKS)

a. Let f be increasing and bounded above on (a, b) with least upper bound L. Prove that f(x) $\to$ L as x $\to$ b^{-}.

(6 marks)

b. State what is meant by " a function f is continuous at c on an interval , a < c < b. How then can discontinuity arise at c? Classify the type of discontinuity in each case.
(7 marks)

QUESTION 4 (13 MARKS)

a. Define a convergent sequence, and a Cauchy sequence. (2 marks)
b. Prove that any convergent sequence is a Cauchy sequence. (5 marks)
c. Show that every Cauchy sequence is bounded. (6 marks)

QUESTION 5 (13 MARKS)

a. Let f: (0, ∞) $\to$ R be defined by

f(x) = $\frac{1}{x}$ (x > 0). Discuss how this function is bounded. Does it attain any maximum and/or minimum? If so, where?
(3 marks)

b. Let f: [0, 1] $\to$ [0, 1] be defined by

f(x) = $\frac{1-x}{1+x}$, (0 ¿ x ≤ 1)

and let g: [0, 1] $\to$ [0, 1] be defined by

g(x) = 4x(1 – x), (0 ≤ x ≤ 1).

Find a formula for f $\circ$ g and g $\circ$ f and show that these functions are not the same.

Show that f^{-1} exists but that g^{-1} does not exist. Find a formula for f^{-1}. (10 marks)

QUESTION 6 (13 MARKS)

Let f be defined on an interval (a, b) except possibly at a point $\xi \in$ (a, b). Demonstrate that

$f(x) \to l$ as $x \to \xi$ if and only if $f(x) \to l$ as $x \to \xi^{-}$ and $f(x) \to l$ as $x \to \xi^{+}$. (13 marks)

QUESTION 7 (13 MARKS)

a. If F is a countable collection of disjoint sets, say $F = \{A_1, A_2, \ldots\}$ such that each set A_n is countable, show that the union $\bigcup_{k=1}^{\infty} A_k$ is also countable. (6 marks)

b. Define an open set and prove that the union of any collection of open sets is an open set. (7 marks)