(Knowledge for Development)

KIBABII UNIVERSITY COLLEGE

A CONSTITUENT COLLEGE OFMASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS

2014/2015 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION

COURSE CODE: MAT 404

COURSE TITLE: **DIFFERENTIAL TOPOLOGY**

DATE: 29/4/15 **TIME:** **8.00AM -10.00AM**

INSTRUCTIONS TO CANDIDATES

Answer Question One in and Any other TWO Questions

TIME: 2 Hours

QUESTION 1 (30 MARKS)

a) Prove that the circle $S^1 = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$ is a one-dimensional manifold. (8marks)

b) Show that if M is compact and $y \in N$ is a regular value of f then $f^{-1}(y)$ is a finite set. (6 marks)

c) Define a manifold (2marks)

d) Suppose $Z = f^{-1}(y)$ for a regular value y of the mapping $f : X \to Y$. Prove that $Ker[df_x : T_x X \to T_y Y] = T_x Z$ at any point $x \in Z$. (8marks)

e) Prove that any point in a smooth manifold M has an open neighborhood in M which is diffeomorphic to an open subset of $\mathbb{R}^n$. (6marks)

QUESTION 2 (20 MARKS)

a) Show that $X \times Y \subset \mathbb{R}^M \times \mathbb{R}^N$ is a smooth manifold (8marks)

b) Show that the tangent space $T_x(M)$ has the same dimension as the smooth manifold M. (6marks)

c) State local immersion theorem without prove. (2marks)

d) Suppose that $f : X \to Y$ is a diffeomorphism and $df_x : T_x(X) \to T_{f(x)}(Y)$. Prove that the dimensions of the two manifolds are same. (4marks)

QUESTION 3 (20 MARKS)

a) Show that an embedding $f : X \to Y$ maps X diffeomorphically into a submanifold of Y. (10marks)

b) Show that every $k-$dimensional manifold admits a one-to-one immersion in $\mathbb{R}^{2k+1}$ (6marks)

c) Show that a proper map $\rho : X \to \mathbb{R}$ exists on any manifold X (4marks)

QUESTION 4 (20 MARKS)

a) Show that the tangent vector with respect to parametrization ¨ is also the tangent vector with respect to parametrization φ'. (6marks)

b) Show that any smooth map f of the closed unit ball $D^n \subset \mathbb{R}^n$ into itself has a fixed point.(8marks)

c) Let M be a compact manifold with boundary. Prove that there does not exist a smooth map $f : M \to \partial M$ that leaves every point of the boundary fixed. (6marks)

QUESTION 5 (20 MARKS)

a) Let $f : M \to N$ be an imbedding, where M is a (non-empty) compact $n-$dimensional smooth manifold and N is a connected $n-$dimensional smooth manifold. Show that f is a diffeomorphism. (4marks)

b) Show that the orthogonal group O(n) is a Lie group (8marks)

c) State the Inverse function theorem without prove. (2marks)

d) Suppose that $f : X \to Y$ is a submersion at x, and $y = f(x)$. Prove that there exists coordinate around x and y such that $f(x_1, \ldots, x_k) = [x_1, \ldots, x_l]$ on Neighbourhoods $N(x)$ and $M(y)$. (6marks)