(Knowledge for Development)

KIBABII UNIVERSITY COLLEGE

A CONSTITUENT COLLEGE OFMASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS

2014/2015 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **MAT 324**

COURSE TITLE: **NUMERICAL ANALYSIS II**

DATE: 30/4/15 **TIME: 3.00PM -5.00PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One in and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (COMPULSORY) (30MARKS)

(a) Obtain a second degree polynomial approximation to

$$f(x) = (1+x)^{\frac{1}{2}}, x \in [0, 0.1],$$

using the Taylor series expansion about $x = 0$. Use the expansion to approximate $f(0.05)$and bound the truncation error. (7marks)

(b) Find the universe of the matrix,

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

using the iterative method, given that its approximate inverse is

$B = \begin{bmatrix} 1.8 & -0.9 \\ -0.9 & 0.9 \end{bmatrix}$. Perform two iterations. (4marks)

(c) The system equation $\boldsymbol{A} \quad = \mathbf{b}$ is said to be solved iteratively by

$x_{n+1} = Mx_n + b$

Suppose $A = \begin{bmatrix} 1 & k \\ 2k & 1 \end{bmatrix}$, $k \neq \sqrt{2}/2$, k is real.

Find a necessary & sufficient condition on k for convergence of the Jacobi method. (4marks)

(d) (i) Use the Euler method to solve numerically the initial value problem.

$u' = -2tu^2$, $u(0) = 1$,

With $h = 0.2$ on the interval $[0, 1]$. (4marks)

(ii) Determine the percentage relative error at $t = 1$ (4marks)

(e) Evaluate the integral

$$\int_0^1 \frac{d}{1+x}$$

Using Gauss-Legendre three point formula. (5marks)

(f) Obtain the Chebyshev linear polynomial approximation to the function

$f(x) = x^3$, on $[0, 1]$ (6marks)

QUESTION TWO (20 MARKS)

Consider the system equation

$$\begin{bmatrix} 1 & -a \\ -a & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

where a, b_1 and $b_2$are non zero real constants.

(a) For which values of a, does the Jacobi and Gauss – Seidel method, converge

(8marks)

(b) Show that the Gauss – Seidel method is atleast twice as fast as the Jacobi method (2marks)

(c) For $a = 0.5$, find the value of ω, the relaxation parameter which minimizes the spectral radius of the SOR iteration matrix (10marks)

QUESTION THREE (20 MARKS)

(a) Derive Taylors formula with Lagrange reminder term (8marks)

(b) Given the Initial value problem.

$$u' = t^2 + u^2 \, , u(0) = 0$$

(i) Determine the first three non-zero terms in the Taylor series for $u(t)$ and hence obtain the value $u(1)$. (10marks)

(ii) Determine time t when the error in $u(t)$ obtained from the first two non-zero terms is said to be less than 10^{-6} after rounding off. (2marks)

QUESTION FOUR (20 MARKS)

(a) Find a uniform polynomials approximation of degree four or less to e^x on $[0, 1]$ using *Lanczos economization* with tolerance of ∈ = 0.01 (8marks)

(b) Obtain the least sequence polynomial approximation of degree one and two for $f(x) = x^{\frac{1}{2}}$ on [o, 1]. (12marks)

QUESITION FIVE (20 MARKS)

(a) What is a spline function of degree n with knots,

$$x_i, i = 0, 1, \dots \dots, n.$$

(4marks)

(b) Derive the natural cubic spline function for $f(x)$. (10marks)

(c) Obtain the natural cubic spline approximation for the function given in tabular form and $m(0) = 0, m(3) = 0$ (6marks)

x	0	1	2	3
$f(x)$	1	2	33	244