(Knowledge for Development)

KIBABII UNIVERSITY COLLEGE

A CONSTITUENT COLLEGE OFMASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS

2014/2015 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **MAT 222**

COURSE TITLE: **CALCULUS III**

DATE: 30/4/15 **TIME: 3.00PM-5.00PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One in and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30marks)

(a) (i) Using first principles determine $f_x(x, y)$,of $f(x, y) = x^2y + 4xy^2 - 3x \quad + 2$. (3mks)

(ii) Given $f(x, y) = xe^{x^2y}$ find f_y at $(1, I\!: 2)$. (2mks)

(b) (i) State the necessary sufficient conditions for the existence of etrema. (2mks)

(ii)Locate and identify etremum for $f(x, y) = (x - 1)(y - 1)$ (2mks)

(c) (i) Define the Lagrange's multipliers. (2mks)

(ii) Find the stationery values of $x^2 + y^2 + z^2$given that $a \quad + b \quad + c \quad = p$. (4mks)

(d) Show that the series $\sum_{n=1}^{\infty} u_n$ is convergent if $\lim_{n\to\infty} u_n = 0$. (3mks)

(e) Use comparison test to show that the series $\sum_{n=1}^{\infty} \frac{1}{2+5^n}$ is convergent. (2mks)

(f) Show that if an infinite series $\sum u_n$ is absolutely convergent then it is convergent. (4mks)

(g) (i) Define an improper integral (1mks)

(ii) Use the domination test to determine whether or not the the integral $\int_0^{\infty} \frac{e \quad x}{1+x^2} d$ converges. (3mks)

(h) Show that a positive term series $\sum_n u_n$ is convergent if and only if its sequence of partial sums $\langle u_n \rangle$ is bounded above. (4mks)

QUESTION TWO (20marks)

(a) (i)State and prove the Weierstress theorem (M-test) for uniform convergence. (6mks)

(ii)Using the Weierstress M-test show that $\sum f_n(x) = \sum \frac{c}{n^2}$ converges uniformly.(2mks)

(b) (i) Show that the series $\sum_{n=1}^{\infty} x_n$is convergent if and only if for each $\varepsilon > 0$, there exists a positive integer m such that $|x_{m+1} + x_{m+2} + \cdots x_n| < \varepsilon, \forall n \geq m$. (4mks)

ii) Show that the series $1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$ is not convergent. (4mks)

(c) (i) Define a power series (1mk)

ii) Find all values of x for which the power series $\sum \frac{n}{5^n} x^n$ is absolutely convergent. (3mks)

QUESTION THREE (20 marks)

a) (i) Define an implicit function. (1mk)

(ii) Find $\frac{\partial}{\partial}$ and $\frac{\partial}{\partial}$ given $x^2y + y^2z + x \ = 0.$ (2mks)

b) Find the equation of the tangent plane to the paraboloid $Z = 1 - \frac{1}{1}(x^2 + 4y^2)$ at the point $(1,1,\frac{1}{2})$ (4mks)

c) Find $\frac{\partial}{\partial}$ and $\frac{\partial}{\partial}$ given $f(x, y) = 4x^2 + xy^2$ where $x = r + s$ and $y = r - s$ (4mks)

d) Find the tangent plane and normal line to the surface $f(x, y, z) = x^2 + y^2 + z^2 - 4 = 0$ at the point $(1,1,\sqrt{2})$. (3mks)

e) Find the extrema for $f(x, y) = s$ in the domain $0 \leq x \leq \pi, 0 \leq y \leq \pi$. (4mks)

f) Find the slope of the surface given by $f(x, y) = -\frac{x^2}{2} - y^2 + \frac{2}{8}$ at the point $(\frac{1}{2}, 1, 2)$ in the x-direction and in the y-direction. (2mks)

QUESTION FOUR (13MKS)

(a) Determine whether or not the following integrals converge

(i) $\int_a^{\infty} \frac{1}{x} d \ , a \geq 0$ (ii) $\int_a^{\infty} \frac{1}{x^2} d \ , a > 0.$ (4mks)

(b) Evaluate $\int_{-\infty}^{+\infty} \frac{1}{x^2+1} d$

(3mks)

(c) Use μ-test to determine whether the following improper integrals are convergent or divergent

i) $\int_a^{\infty} \frac{d}{x^{\frac{1}{2}}(1+x^{\frac{1}{2}})}, a > 0$ ii) $\int_0^{\infty} \frac{x}{(1+x)^3} d \ , a > 0$ iii) $\int_a^{\infty} \frac{x^{2m}}{1+x^{2n}} d \ , a > 0.$ (7mks)

(d) i) State the Dirichlet's test for convergence for an improper integral. (2mks)

ii) Test for the convergence of $\int_a^{\infty} \frac{s}{\sqrt{x}} d \ , a > 0$ using the Dirichlet's test. (3mks)

(e) Find the Cauchy's principal value of $\int_0^1 \frac{1}{x^{\frac{2}{3}}} d$ (2mks)

QUESTION FIVE (20 MKS)

(a) Evaluate $\iint_R f(x, y) d$ for $f(x, y) = 1 - 6x^2y$ and $0 \leq x \leq 2, -1 \leq y \leq 1.$ (2mks)

(b) Evaluate $\iint_R \frac{s}{x} d$ where R is the triangle in the x plane bounded by the x-axis , the line $y = x$ and the line $x = 1$. (3mks)

(c) Find the area of the region bounded by $y = 2x^2$ and $y^2 = 4x$. (4mks)

(d) Find the average value of $f(x, y) = x$ over the rectangle

$R: 0 \leq x \leq \pi, 0 \leq y \leq 1$. (4mks)

(e) i) Define conditional convergence for improper integrals. (2mks)

ii)Check for convergence, conditional convergence or divergence for $\int_0^\infty f(x)d$ where $f(x)$

is defined as follows: $f(x) = \begin{cases} 1, & 0 \leq x < 1 \\ -\frac{1}{2}, & 1 \leq x < 2 \\ \frac{1}{3}, & 2 \leq x < 3 \\ -\frac{1}{4}, & 3 \leq x < 4 \\ \left(-\frac{1}{n}\right), & n-1 \leq x < n \end{cases}$ (5mks)