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**UNIVERSITY EXAMINATIONS**

**2012 /2013 ACADEMIC YEAR**

**FOR THE DEGREE OF BACHELOR**

**OF SCIENCE (MATHEMATICS)**

COURSE CODE: **MAT 202**

COURSE TITLE: **LINEAR ALGEBRA II**

**DATE:** 20th August 2013 **TIME:** 9.00am – 2.00pm

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***INSTRUCTIONS***

*This paper consists of **TWO** sections; **A** and **B**. Answer **BOTH** questions in **SECTION A** and **ANY OTHER THREE** questions from **SECTION B**.*

## SECTION A

*Answer **BOTH** questions in this section.*

**QUESTION 1 (16 marks)**

a. Prove that if A is an orthogonal matrix, then $|A| = \pm 1$. (3 marks)

b. State what is meant by the terms eigenvalue and eigenvector
What is the characteristic polynomial of a matrix A? (3 marks)

c. Find the distance of the point **x** = (4, 1, -7) of $\mathbf{R}^3$ from the subspace W consisting of all vectors of the form (a, b, b) (5 marks)

d. Write the following quadratic form in terms of matrices.
$$-3x^2 - 7xy + 4y^2$$ (1 marks)

e. Consider the bases B = {(1, 2), (3, -1)} and B' = {(3, 1), (5, 2)} of $\mathbf{R}^2$. Find the transition matrix from B to B'. If **u** is a vector such that $\mathbf{u}_B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, find $\mathbf{u}_{B'}$. (4 marks)

**QUESTION 2 (15 MARKS)**

a. Define each of the following:
Invariant subspace
Linear functional
Dual space
(3 marks)

b. Consider the operator $T(x, y) = (2x, x + y)$ on $\mathbf{R}^2$. Find the matrix of $T$ with respect to the standard basis $B = \{(1, 0), (0, 1)\}$ of $\mathbf{R}^2$. Use the transformation $A' = P^{-1}AP$ to determine the matrix A' with respect to the basis B' = {(-2, 3), (1, -1)}. (6 marks)

c. Let $\mathbf{u} = (x_1, x_2)$ and $\mathbf{v} = (y_1, y_2)$ be elements of $\mathbf{R}^2$. Prove that the following defines an inner product on $\mathbf{R}^2$.
$$\langle \mathbf{u}, \mathbf{v} \rangle = 4x_1y_1 + 9x_2y_2$$ (4marks)

d. Consider he vector space $P_n$ of polynomials with inner product
$$\langle f, g \rangle = \int_0^1 f(x)\, g(x)\, dx \, .$$
Determine the norm of the function $f(x) = 3x^2 + 2$. (2 marks)

## SECTION B

*Answer ANY THREE questions from this section.*

QUESTION 3 (13 MARKS)

a. Solve the difference equation $a_n = 2a_{n-1} + 3a_{n-2}$, for n = 3, 4, 5, . . . (3 marks)
with initial conditions $a_1 = 0$, $a_2 = 1$. Use the solution to determine $a_{13}$.
b. Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix}$$

(7 marks)

Determine the basis and the dimension of each eigenspace associated with this matrix.
(3 marks)

**QUESTION 4 (13 MARKS)**

Analyze the following equation. Sketch its graph.

$$6x^2 + 4xy + 9y^2 - 20 = 0$$ (13 marks)

**QUESTION 5 (13 MARKS)**

a. What does it mean to say that a matrix is orthogonally diagonalizable? (1 mark)
b. Orthogonally diagonalize the symmetric matrix

$$A = \begin{bmatrix} 1 & 5 \\ 5 & 1 \end{bmatrix}$$ . Give the similarity transformation.

c. Compute $A^8$.
d. Prove that similar matrices have the same eigenvalues.

**QUESTION 6 (13 MARKS)**

Consider the following matrix

$$A = \begin{bmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{bmatrix}.$$

a. What type of matrix is the matrix A? Determine all the eigenvalues of A.
b. Determine the eigenspaces of A. What relationship exists between the dimension of eigenspaces and the eigenvalues?
c. Are the eigenvectors linearly independent? Explain.
d. Are the eigenspaces orthogonal? Justify your answer. (13 marks)

**QUESTION 7 (13 MARKS)**

Consider the linear operator $T(x, y) = (3x + y, x + 3y)$ on $\mathbf{R}^2$. Find a diagonal matrix representation of $T$. Determine the basis for this representation and give a geometrical interpretation of T. (13 marks)