(K*nowledge for Development)*

KIBABII UNIVERSITY COLLEGE

A CONSTITUENT COLLEGE OF

MASINDE MULIRO UNIVERSITY OF

SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS

2014/2015 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF

BACHELOR OFSCIENCE COMPUTER SCIENCE

COURSE CODE: **CSC 122**

COURSE TITLE: **DISCRETE STRUCTURES**

DATE: 7th MAY, 2015 **TIME**: 8.00 - 10.00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One in Section A and Any other TWO (2) Questions in Section B

INSTRUCTIONS

Answer questions one and any other two questions only.

This paper consist of 3 printed pages

Question one carries 30 marks and the other questions carry 20 marks each

Question One (30 Marks)

(a) What is the largest possible number of vertices in a data storage path graph with 70 edges and all vertices of degree at least 3 (4 marks)

(b) Assume that a country with currently 100 million people has a population growth rate of 1% per year, and it also receives 100 thousand immigrants per year. Find its population in 10 years from now. (Assume that all the immigrants arrive in a single batch at the end of the year) (6 marks)

(c) (i) Consider the recurrence relation $a_n = 2a_{n-1} - a_{n-2}$ for $n = 2,3,4,$ ... is the sequence $\{a_n\}$ with $a_n = 3n$ a solution of this recurrence relation? (2 marks)

(ii) What is the solution of the recurrence relation $an = 8a_{n-1} - 16a_{n-2}$ with $a_0 = 1$ and $a_1 = 6$? (6 marks)

(d) Write formally the statement "for every real number there is a greater real number". Write the negation of that statement. (4 marks)

(e) Let $P(x)$ be a statement "$(x+1)^2 = x^2 + 2x + 1$". If the universal discourse consists of the integers, what is the truth value of:

(i) $\forall x \ (x)$ (1 mark)

(ii) $\exists X \ P(x)$ (1 mark)

(f) Define the terms *path* and *loop* as used in graph theory (2 marks)

(g) Given that $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ evaluate A ⊙ B (3 marks)

Question Two (20 Marks)

(a) Translate into English the predicate logic used in artificial language $\exists x(P(x) \Lambda \neg Q(x))$, where the P(x) says that x is a bird and Q(x) says that x can fly. Take the universe of discourse to be all living creatures. Suggest a witness to the existential quantifier (that is, a creature that makes the sentence true). (2 marks)

(b) (i) Write in math notation the following English sentence: "Every number is divisible by 2 or by 3" (use d|n for "n is divisible by d"). (4 marks)

(ii) For which universe of discourse is it true? (1 mark)

(iii) For which universe of discourse is it false? (1 mark)

(iv) State it is true or false if the universe of discourse complex numbers (1 mark)

(c) Assign symbols to represent the predicates and write the following in symbolic form supplying the domain of discourse. "**Every political party has its years**" (5 marks)

(d) Form a binary search tree for the data 16, 24, 7, 5, 8, 20, 40 and 3 in the given order (6 marks)

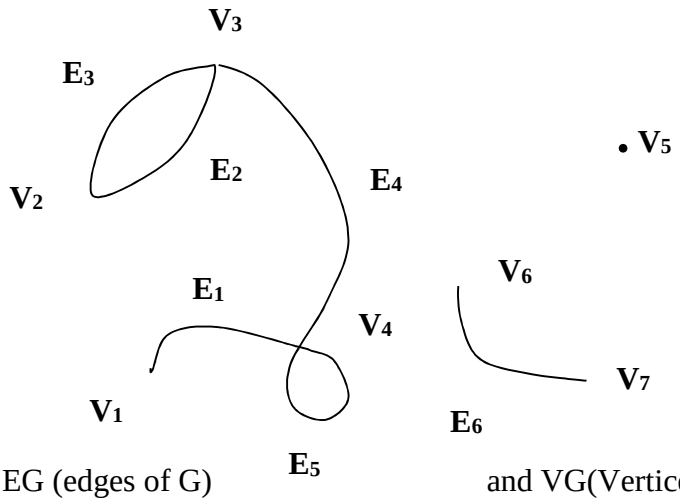
Question Three (20 Marks)

(a) Consider an undirected simple graph that has vertices a, b, c, d, and e of degree 4,3,3,2,2.

(i) How many edges does it have? (3 marks)

(ii) Draw this graph. (5 marks)

(b) Consider the following graph G



i. Find EG (edges of G) and VG(Vertices of G) (2 marks)

ii. List the isolated vertices. (1 mark)

iii. List the loops. (1 mark)

iv. List the parallel edges. (1 mark)

(c) Suppose that a tree T has N_1 vertices of degree 1, 2 vertices of degree 2, 4 vertices of degree 3 and 3 vertices of degree 4. Find N_1 (7 Marks)

Question Four (20 Marks)

(a) The Kenya Bureau of standards discovered that there were fake microchips being sold in the market and hence decided to be testing all microchips before accepting them. The test machines have not been calibrated for some time hence at times give faulty results. The result is recorded as positive if the test machine gives it as genuine. From the sample to be tested the probability of picking a genuine microchip is 0.9 and it testing negative is 0.05. The probability of a fake microchip test positive from the sample is 0.2.

(i) Illustrate the probability of the test results by using a tree diagram (3 marks)

(ii) Find the probability that the test results is positive (3 marks)

(iii) Find the probability of the test being correct (3 marks)

(iv) what is the probability that microchip testing positive is genuine (5 marks)

(b) A problem in discrete structures II is given to three students whose chance of solving it are ½, ⅓, and ¼ respectively. What is the probability that at least one of them solves it correctly (6 marks)

Question Five (20 Marks)

(a) Study the recurrence relation given below and answer the questions that follow

$$\begin{aligned} a_1 &= 0 \\ a_n &= \left(1 - \frac{1}{n}\right) a_{n-1} + 2n, \text{ for } n \geq 2. \end{aligned}$$

(i) Find the values of and a_2, a_3 (2 marks)

(ii) If $b_n = na_n$, Show that $b_n = b_{n-1} + 2n^2$ for $n \geq 2$. (3 marks)

(iii) Solve the recurrence (9 marks)

$$\begin{aligned} b_1 &= 0 \\ b_n &= b_{n-1} + 2n^2, \text{ for } n \geq 2. \end{aligned}$$

(b) Ngurwe, Wangwe and Bombo went to buy punched cards, magnetic tapes and flash disks in a Nakumatt chain of supermarkets. Ngurwe bought two cards, one tape and three disks for Kshs. 3500.00. Wangwe bought two cards, two tapes and

four disks for Kshs 4900.00. Bombo bought two cards, one tape and two disks for Kshs 2500.00. Find the cost price of each of the storage devices. (6 marks)