KIBABII UNIVERSITY COLLEGE (KIBUCO)

MAIN CAMPUS

UNIVERSITY EXAMINATIONS
2014 /2015 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER EXAMINATIONS

MAIN EXAMINATION

FOR THE DEGREE

OF

BACHELOR OF EDUCATION SCIENCE

COURSE CODE: STA 241

COURSE TITLE: STATISTICS AND PROBABILITY

DATE: Tuesday 20th January 2015 **TIME: 8.00-10.00 a.m.**

INSTRUCTIONS TO CANDIDATES:`1

Answer Question ONE and any other Two Questions

TIME: 2 Hours

QUESTION 1: (30 marks)

(a) For a discrete random variable, X, define the following terms;

(i) Expected value, E(X)

(ii) Moment generating function

(iii) Characteristic function

(iv) Variance of X, Var(X)

(1+1+1+1 Marks)

(b) Illustrate how you would get the variance of a random variable X from a moment generating function. (3 marks)

(c) A random variable X has the following density function,

$$f(x) = \begin{cases} e^{-x},,,,, x \le 0 \\ 0,,,,, x < 0 \end{cases}$$

Find (i) expected value of X, E(X) (4 marks)

(ii) $E\left(e^{\frac{2X}{3}}\right)$, that is, $E\left(\exp\frac{2X}{3}\right)$ (4 marks)

(iii) the variance of X, Var(X) (5 marks)

(d) Suppose that a game is to be played with a single die assumed fair. In this game a player wins \$ 20 if a 2 turns up, \$ 40 if a 4 turns up; loses \$ 30 if a 6 turns up; while the player neither wins nor loses if any other face turns up.

Find,

(i) The expected sum of money to be won (5 marks)

(ii) The variance of the sum of money to be won (5 marks)

QUESTION 2: (20 marks)

A random variable X is said to have a Binomial distribution if the probability function of X, f(x) is given as

$$P(X = x) = f(x) = \binom{n}{x} p^x (1-p)^{n-x} \qquad x=0,1,2,\ldots\ldots\ldots,n$$

Where p is the probability of success and q=1-p

(a) Show that the moment generating function of X, $M_X(t)$ is given by

$$M_X(t) = (q + pe^t)^n \qquad (\ 7 \text{ marks })$$

Comment on the distribution of X when n=1. (3 marks)

(b) Use the moment generating function in 2(a) above to verify that,

i) the expected value of X, E(X)=np (5 marks)

ii) the variance of X, Var(X)=npq (5 marks)

QUESTION 3: (20 marks)

Let X be a random variable that is poisson distributed with parameter a and moment generating function, $M_X(t)$ given as :

$$M_X(t) = e^{a(e^t - 1)}$$

(a) Use the $M_X(t)$ to find :

i)the expected value of X, E(X) (5 marks)

ii)the variance of X, Var(X) (5 marks)

(b)The average number of calls arriving on a busy office mobile phone is 90 per hour. Obtain the probability of receiving,

i) 2 calls per minute on that mobile phone (4 marks)

ii) Between 1 and 4 calls per minute on the mobile phone (2 marks)

iii) No calls in a minute on the phone (2 marks)

iv) No calls in an hour on that mobile phone (2 marks)

(Assume that the number of calls are poisson distributed)

QUESTION 4: (20 marks)

(a) Define a Standard Normal random variable (2 marks)

Comment on the shape and area under the Standard normal curve (2 marks)

(b) The time taken by the milkman to deliver to the high street is normally distributed with a mean of 12 minutes and standard deviation of 2 minutes. He delivers milk every day. Estimate the number of days during the year when he takes,

i)longer than 17 minutes (3 marks)

ii)less than ten minutes (3 marks)

iii)between nine and thirteen minutes (3 marks)

(c) Let $X \approx N(\text{«}, \pm^{2})$,with a moment generating function given as :

$$M_X(t) = \exp(\text{«}t + \frac{\pm^{2}t^{2}}{2})$$

Derive expressions for E(X) and Var (X) (7 marks)

QUESTION 5: (20 marks)

(a) Consider X to be binomially distributed with probability mass function, that is, P(X=x) as stated in Question 2 above. Establish the validity of the Poisson approximation to the Binomial distribution. (10 marks)

(b) State and prove the Central limit theorem (10 marks)