On Joint Essential Numerical Range of Aluthge Transform

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Abstract

The concept of Aluthge transform was first introduced by A. Aluthge in 1990 in his study of p-hyponormal operators. Properties of the transform have since been investigated by several authors. Some elementary spectral and numerical range properties and related results of Aluthge transform have been obtained. For instance, in 2002, Yuan Wu proved that the closure of the numerical range of Aluthge transform of the operator T is contained in that of T. In 2007, Guoxing Ji, Ni Liu and Ze Li together showed that the essential numerical range of Aluthge transform is contained in the essential numerical range of T: However, the properties of the Aluthge transform have not been exhaustively studied. For instance, the joint essential numerical range of the Aluthge transform has not been studied. This paper will therefore introduce and study the joint essential numerical range of Aluthge transform and establish its properties. In particular, this paper shows that the joint essential numerical range of the Aluthge transform in contained in the joint essential numerical range of an m-tuple operator $T = T_1, \ldots$, Tm. The results of this study will be helpful in the development of the research on numerical ranges and may also be applied by mathematicians in solving several problems in operator theory.