(Knowledge for Development)

KIBABII UNIVERSITY COLLEGE

A CONSTITUENT COLLEGE OFMASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS

2014/2015 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION

COURSE CODE: **MAT 422**

COURSE TITLE: **PARTIAL DIFFERENTIAL EQUATIONS II**

DATE: 28/4/15 **TIME:** **8.00AM -10.00PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One in and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS) (COMPULSORY)

a. The diameter of a semi-circular plate of radius a is kept at 0^0C and the temperature at the semi-circular boundary is T^0C. Determine the steady state of the temperature. (6 marks)

b. Solve $r - s = \cos x \cos 2y$ (5 marks)

c. Use the method of separation of variables to solve $r - 2p - q = 0$ (5 marks)

d. A string is fixed at two points l apart and is stretched. The motion takes place by displacing the string in the form $y = a\sin(\frac{\pi}{l})$ from which it is released at time t=0. Show that the displacement of any point at a distance x from one end at time t is

$y(x,t) = a\sin\left(\frac{\pi}{l}\right)\cos(\frac{\pi}{l})$ (6 marks)

e. Solve $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$ by the method of separation of variables (8 marks)

QUESTION TWO (20 MARKS)

a. An infinitely long plane uniform plate is bounded by two parallel edges and an end at the right angles to them. The breadth is π; the end is maintained at a temperature. Find the temperature at any point of the plate in the steady state (10 marks)

b. Determine the region in the x -plane in which the equation $y\frac{\partial^2 u}{\partial x^2} + x\frac{\partial^2 u}{\partial y^2} = 0$ is elliptic. Find its canonical form in this region (10 marks)

QUESTION THREE (20 MARKS)

a. Solve $\frac{\partial^3 z}{\partial x^3} - \frac{3(\partial^3 z)}{2\partial x^2 \partial} + \frac{4(\partial^3 z)}{\partial y^3} = e^{x+2y}$ (5 marks)

b. A rectangular plate with insulated surfaces is 10 cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature along the short edge $y = 0$ is given by

$$u(x,0) = 20x, \qquad 0 < x \le 5,$$

$$u(x,0) = 20(10 - x), 5 < x < 10$$

While the two long edges $x = 0$ and $x = 10$ as well as the other short edges are kept at 0^0C. Find the steady state temperature at any point (x, y) of the plate. (15 marks)

QUESTION FOUR (20 MARKS)

a. Show that the equation $r - 4s + 4t + p - 2q = 0$ is parabolic. Find its canonical form and hence solve it. (7 marks)

b. Classify the equation $r + t - x + y = 0$ then reduce it to its canonical form and solve it (7 marks)

c. An insulate rod of length L has its ends A and B maintained at 0^0C and 100^0C, respectively until steady-state conditions prevail. If B is suddenly reduced to 0^0C and maintained at 0^0C, find the temperature at a distance x from A at a time t (6 marks)

QUESTION FIVE (20 MARKS)

a. Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$

under the condition

$u = 0$, when $x = 0$ and $x = \pi$

$\frac{\partial}{\partial} = 0$ when $t = 0$ and

$u(x, 0) = x, 0 < x < \pi$ (10 marks)

b. Show how we can solve $\frac{\partial^2 y}{\partial t^2} = C^2 \frac{\partial^2 y}{\partial x^2}$ using the D'Alembert's Method.(10 marks)