(Knowledge for Development)

KIBABII UNIVERSITY COLLEGE

A CONSTITUENT COLLEGE OFMASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS

2014/2015 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION

COURSE CODE: **MAT 304**

COURSE TITLE: **COMPLEX ANALYSIS I**

DATE: 28/4/15 **TIME:** 11.30AM -1.30PM

INSTRUCTIONS TO CANDIDATES

Answer Question One in and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a. Use De Moivre's theorem to show that $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$.
(4 marks)

b. State and prove the necessary condition for a function $f(x, y)$ to be harmonic.
(4 marks)

c. Find the first four terms of the Taylor series expansion for the function $f(z) = \dfrac{1}{(z-1)(z-3)}$ about $z = 4$ and state the region of convergence. (5 marks)

d. Distinguish between conformal and isogonal mapping. (2 marks)

e. Given $v(x, y) = 2xy - \dfrac{y}{x^2 + y^2}$, use the Milne-Thomson method to find a function $u(x, y)$ such that $f(z) = u + iv$ is analytic. (5 marks)

f. Prove that if $f(z)$ is analytic within and on a simple closed curve C and a is any point inside C then $f(a) = \dfrac{1}{2\pi i}\oint_C \dfrac{f(z)}{z-a}dz$ (5marks)

Hence evaluate $\oint_C \dfrac{(z+4)}{(z^2+2z+5)}dz$ where C is $|z+1-i| = 2$ (5 marks)

QUESTION TWO (20 MARKS)

a. Evaluate $\left(\dfrac{\sqrt{3}-i}{\sqrt{3}+i}\right)^4 \left(\dfrac{1+i}{1-i}\right)^5$ (4 marks)

b. Find the residues of $f(z) = \dfrac{z^2 - 2z}{(z+1)^2(z^2+4)}$ at all its poles in the finite plane.
(6 marks)

c. Obtain the Laurent's expansion for the function $f(z) = \dfrac{1}{z^2 - 3z + 2}$ in the region

i. $|z| < 1$ (4 marks)

ii. $1 < |z| < 2$ (3 marks)

iii. $|z| > 2$ (3 marks)

QUESTION THREE (20 MARKS)

a. Given two complex numbers z_1 and z_2 where $|z_1| = r_1, |z_2| = r_2, \arg(z_1) = \theta_1$ and $\arg(z_2) = \theta_2$; show that

i. $|z_1 z_2| = |z_1||z_2|$ (2 marks)

ii. $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$ (2 marks)

b. Evaluate $\int_C \bar{z}dz$ from $z = 0$ to $z = 4 + 2i$ along the curve C given by:

i. $z = t^2 + it$ (3 marks)

ii. the line from $z = 0$ to $z = 2i$ and then the line from $z = 2i$ to $z = 4 + 2i$ (5 marks)

c. find the image of the rectangle whose vertices are $(0,0), (1,0), (1,2), (0,2)$ by means of the linear transformation $w = (1+i)z + 2 - i$. Also sketch the image. (6 marks)

d. Evaluate $\lim\limits_{z \to \frac{i}{2}} \dfrac{(2z-3)(4z+i)}{(iz-1)^2}$ (2 marks)

QUESTION FOUR (20 MARKS)

a. Solve the equation $z^2 + (2i - 3) + 5 - i = 0$.

(7 marks)

b. State and prove the Cauchy-Goursat theorem.

(5 marks)

c. Show that $v(x, y) = e^x (x \cos y - y \sin y)$ is a harmonic function and find the analytic function for which $e^x (x \cos y - y \sin y)$ is the imaginary part. (8 marks)

QUESTION FIVE (20 MARKS)

a. Find the bilinear transformation which maps the points $z_1 = 2, z_2 = i, z_3 = -2$ onto the points $w_1 = 1, w_2 = i$ and $w_3 = -1$. (4 marks)

b. Express the function $f(z) = \dfrac{1-z}{1+z}$ in the form $u + iv$. (3 marks)

c. Prove Laurent's theorem: If $f(z)$ is analytic inside and on the boundary of a ring shaped region R bounded by two concentric circles C_1 and C_2 with centre at a and respective radii r_1 and r_2 $(r_1 > r_2)$, then for all z in R, $f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n + \sum_{n=1}^{\infty} a_{-n} (z-a)^{-n}$

where $a_n = \dfrac{1}{2\text{®}i} \oint_{C_1} \dfrac{f(w)}{(w-a)^{n=1}} dw, \; n = 0,1,2....$ and $a_{-n} = \dfrac{1}{2\text{®}i} \oint_{C_2} \dfrac{f(w)}{(w-a)^{-n+1}} dw, \; n = 1,2,3......$

(13 marks)