(Knowledge for Development)

KIBABII UNIVERSITY COLLEGE

A CONSTITUENT COLLEGE OFMASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS

2014/2015 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION

COURSE CODE: MAT 204

COURSE TITLE: REAL ANALYSIS I

DATE: 29/4/15 **TIME:** 11.30AM -1.30PM

INSTRUCTIONS TO CANDIDATES

Answer Question One in and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Show that if $x \neq 0$, then $x^{-1} \neq 0$ and x^{-1} is unique. (3mks)

b) For every $x \neq 0$, show that $x^2 > 0$, hence show that $1 > 0$. (3mks)

c) Let (S, <) be an ordered set and E a subset of S, if the least upper bound of E (lubE) and the greatest lower bound of E (glbE) exist. Show that i) the lubE is unique (4 mks)

ii) the glb.E is unique. (4 mks)

d) Show that $\sqrt{3}$ is an irrational number. (4mks)

e) State the completeness axiom for $\Re$ (2mks)

f) Let A be a nonvoid subset of $\Re$ which is bounded above. Define a set

B by $B = \{-x; x \in A\}$, show that B is bounded below and

–sup A= inf B. (4mks)

g) If a and b are given real numbers such that for every real number

$£ > 0,\ a \leq b + £$, show that $a \leq b$ (5mks)

h) Define an inductive set? (2mks)

QUESTION TWO (20 MARKS)

a) For any subset E of a metric space $(X, \circ)$, prove that E^0 is an open set.

(6mks)

b) Consider the metric space $(\Re, d)$ and let $f : \Re \to \Re$ be defined by $f(x) = |x|$.
Show that f is uniformly continuous. (6mks)

c) Show that the limit of a convergent sequence is unique in a metric space (8mks)

QUESTION THREE (20 MARKS)

a) Show that every infinite set E contains a countable subset A. (7mks)

b) Differentiate between an algebraic and a transcendental number giving examples in each case (3mks)

c) Does the equation $x^2+1=0$ have a solution in $\Re$? Show your working. (4mks)

d) Define the following terms;

i. A metric space (4mks)

ii. An interior point of a set E (2mks)

<u>QUESTION FOUR (20 MARKS)</u>

a) Suppose that an open interval $(0,1)$ is equivalent to $\Re$. Show that $\Re$ is uncountable (10mks)

b) State and provide a proof of Cauchy –Schwarz inequality. (10mks)

<u>QUESTION FIVE (20 MARKS)</u>

a) Let A and B be nonvoid subsets of $\Re$ and define the set $A+B=\{x+y; x\in A, y\in B\}$,

b)

i. If A and B are bounded above, then show that A+B is also bounded above and sup(A+B)=sup.A+sup.B

ii. (5mks)

iii. If A and B are bounded below, then show that A+B is also bounded below and inf.(A+B)=inf.A+inf. B (5mks)

c) For every real numbers x and a, $a>0$, show that $|x|\leq a$ iff $x\in[-a,a]$ (4mks)

d) Let A, B, C be nonvoid sets and $f:A\to B$ and $g:B\to C$ be bijections. Then, prove that $(g\circ f)^{-1}$ exists and $(g\circ f)^{-1}=f^{-1}\circ g^{-1}$. (6 mks)