(Knowledge for Development)

KIBABII UNIVERSITY COLLEGE

A CONSTITUENT COLLEGE OFMASINDE MULIRO UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS

2014/2015 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

(COMPUTER SCIENCE)

COURSE CODE: **MAT 201**

COURSE TITLE: **LINEAR ALGEBRA I**

DATE: 29/4/15 **TIME:** 11.30AM -1.30PM

INSTRUCTIONS TO CANDIDATES

Answer Question One in and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

(a) (i)Define the terms subspace of vector spaces and a basis of a vector space (2mks)

(ii) Determine whether $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(x_1 \quad x_2) =$ $[x_2, \quad x_1 - \quad x_2, \quad 2x_1 + \quad x_2]$ is a linear transformation. (5mks)

(b) Determine which of the matrices $= \begin{bmatrix} 1 & 3 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 4 & 0 \\ 1 & 3 & 2 \\ 0 & 0 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & -1 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}$,

and $D = \begin{bmatrix} 1 & 3 & 2 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ are in achelon form. (4mks)

(c) Solve the system below using Gaussian Elimination with back-substitution. (4mks)

$$b - 3c = -5$$
$$2a + 3b - c = 7$$
$$4a + 5b - 2c = 10$$

(d) (i) Show that the vector $\vec{u} = (a, \quad 2a)$ in $\mathbb{R}^2$ is a subspace of $\mathbb{R}^2$ (4mks)

(ii) Determine whether the set of vectors below is linearly independent or dependent (5mks)

$$S = \left\{ \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} \right\}$$

(e) (i) Find the kernel of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ represented by $T(x_1, x_2) = (x_1 - 2x_2, 0, -x_1)$. (3mks)

(ii) Determine whether the vector $b = [1, \quad -7, \quad -4]$ is in the span of vectors $v = [2, \quad 1, \quad 1]$ and $w = [1, \quad 3, \quad 2]$ (3mks)

QUESTION TWO (20MARKS)

(a) Use Gauss – Jordan elimination to solve the system

$$x+2y-2z = -3$$
$$w + 2x - y = 2$$
$$2w + 4x + y - 3z = -2$$
$$w - 4x - 7y - z = -19$$

(6mks)

(b) Write the vector w = (1, 1, 1) as a linear combination of vectors in the set S, where S = {(1, 2, 3), (0, 1, 2), (-1,0,1)} (7mks)

(c) If A is a matrix given by $A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \end{bmatrix}$ write down

i. Row space
ii. Column space
iii. Null space
Of the matrix A, above. (4mks)

(d) Proof that if W_1 and W_2 are subspaces of V then so is $W_1 \cap W_2$. (3 mks)

QUESTION THREE (20 MARKS)

(a) Find the rank, a basis for the row space ,a basis for the column space and a basis for the null space of the matrix B given by $B = \begin{bmatrix} 1 & 3 & 0 & -1 & 2 \\ 0 & -2 & 4 & -2 & 0 \\ 3 & 11 & -4 & -1 & 6 \\ 2 & 5 & 3 & -4 & 0 \end{bmatrix}$ (9 mks)

(b) Determine which of the two subsets is a subspace of $\mathbb{R}^2$

i) The set of all points on the line $x + 2y = 0$
ii) The set of points on the line $x + 2y = 1$

(5mks)

(c) Proof that:
(i) If A is an invertible matrix, then its inverse is unique.(3mks)

(ii) If A, B and C are invertible matrix and AC = BC , then A = B (3mks)

QUESTION FOUR (20 MARKS)

(a) Determine whether the set of vectors S in $\mathbb{R}^3$ is linearly independent or linearly dependent. Where, S = {(1,2,3), (0,1,2), (-2,0,1)} (6mks)

(b) Find the inverse of the matrices below (6 mks)

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 5 & -3 \\ -3 & 2 & -4 \end{bmatrix}$$

(c) Express the solution set of the homogenous system

$$x_1 - 2x_2 + x_3 - x_4 = 0$$
$$2x_1 - 3x_2 + 4x_3 - 3x_4 = 0$$
$$3x_1 - 5x_2 + 5x_3 - 4x_4 = 0$$
$$-x_1 + x_2 - 3x_3 + 2x_4 = 0$$

as a span of solution vectors . (4 mks)

(d) Find the dimension of the subspace W= S $(w_1, \quad w_2, \quad w_3, \quad w_4)$ of $\mathbb{R}^3$ where $w_1 = [1 \quad -3 \quad 1]$. $w_2 = [-2 \quad 6 \quad -2]$ $w_3 = [2 \quad 1 \quad -4]$ $w_4 = [-1 \quad 10 \quad -7]$ (4mks)

QUESTION FIVE (20 MARKS)

(a) Solve the system

$$x - 2y + 3z = 9$$
$$-x = 3y = -4$$
$$2x - 5y + 5z = 17$$

using Gauss- Jordan elimination method (7mks)

(b) For the set of vectors in $m_{2,2}$. The set

(c) S = $\left\{\begin{bmatrix} 0 & 8 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -2 & 0 \\ 1 & 3 \end{bmatrix}\right\} = \{\overrightarrow{v_1} \quad \overrightarrow{v_2} \quad \overrightarrow{v_3} \quad \overrightarrow{v_4}\}$. Express $\overrightarrow{v_1}$ as a linear combination of the vectors $\overrightarrow{v_2}, \overrightarrow{v_3},$ and $\overrightarrow{v_4}$ (6mks)

(d) Find the kernel of the linear transformation, $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$

defined by $T(x) = A(x)$, where

A = $\begin{bmatrix} 1 & -1 & -2 \\ -1 & 2 & 3 \end{bmatrix}$ (7 mks)